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INTRINSIC SCALES IN THERMOHYDROGASDYNAMICS

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Abstract—Intrinsic scales and similarity criteria are considered as characteristics of local interactions in flows of homogeneous and inhomogeneous mixtures. Thereby in the analysis of similarity an important class of values is separated whose elements have been used previously, as a rule, at an intuitive level.

NOMENCLATURE

c,	specific heat capacity;
C_f,	friction factor;
D,	diffusivity;
G,	mass flow rate with respect to unit width of flowing surface;
g,	acceleration;
i,	enthalphy;
j,	mass velocity;
p,	pressure;
R,	radius;
r,	heat of physicochemical conversion;
q,	heat flux density;
T,	temperature;
t,	time;
V, U,	flow velocity;
α,	heat transfer coefficient;
Δ,	difference;
δ,	thickness;
λ,	thermal conductivity coefficient;
μ,	viscosity;
v,	kinematic viscosity;
ρ,	density;
σ,	surface tension.

Subscripts

$'$,	liquid;
$''$,	gas, vapour;
cr,	critical;
w,	wall;
b,	bulk;
h,	thermal;
t,	turbulent.

Similarity factors

$$Ar_* = \frac{\rho'^{1/2}\sigma^{3/2}}{\mu'^2\left[g\left(1-\frac{\rho''}{\rho'}\right)\right]^{1/2}},$$

modified Archimedes number;

$$K = \frac{r}{\Delta i},$$

thermal similarity number for physicochemical conversions accompanied by heat evolution or absorption (introduced by the author elsewhere);

$$K_* = \frac{c'U''p}{g\lambda'},$$

dynamic similarity number for bubbling or boiling (introduced by the author elsewhere);

$$k = \frac{U''_{cr}\rho''^{1/2}}{[\sigma g(\rho'-\rho'')]^{1/4}},$$

structural stability factor for gas–liquid mixture (introduced by the author elsewhere);

$$M_* = \left(\frac{\rho}{\rho}\right)^{1/2}\left(\frac{g\sigma}{\rho'-\rho''}\right)^{1/4},$$

capillary acoustic interaction number (modified Mach number, introduced by the author elsewhere);

$$Nu_* = \frac{\alpha}{\lambda'}\left[\frac{\sigma}{g(\rho'-\rho'')}\right]^{1/2},$$

Nusselt number for boiling (introduced by Jacob);

$$Nu^* = \frac{\alpha}{\lambda'}\left[\frac{v'^2}{g[1-(\rho''/\rho')]}\right]^{1/3},$$

condensation Nusselt number (introduced by Colburn);

$$Pe_* = \frac{c'j''_w}{\lambda'}\left[\frac{\sigma}{g(\rho'-\rho'')}\right],$$

bubbling or boiling Peclet number (introduced as dynamic scale for mass vapour phase velocity by Rosenov);

$$Pr = \frac{v}{a},$$

Prandtl number;

$$Re_f = \frac{G'}{\mu'},$$

liquid film Reynolds number;

$$St = \frac{\alpha}{c\rho U},$$

Stanton number.

INTRODUCTION

A SIMILARITY analysis involves combinations of physical properties of the medium and its dynamic scales having dimensions of length, velocity, time and so on. These derived values can be considered as scale parameters of propagations of one or any other perturbations inside a given system. Their independence of the boundary conditions or unambiguous relations permit us to construct universal similarity numbers characterizing local interactions inside the object under consideration.

Deliberate operation with intrinsic scales of the process makes the similarity analysis more distinct, it is efficient to clarify self-similar processes by one or the other parameters, promotes elaborating a compact algorithm to compose the similarity numbers typical of a given specific process.

Certainly, a reader in a cynical mood could characterize the content of this present paper by the aphorism: "It might be of interest, but it seems to me that much is already known." This may be true; but at the same time I believe this is far from being so. The similarity analysis, despite the apparent simplicity of its conceptions, has no standard algorithms and hence requires the research worker's intuition and skill in operating with values, dimensions and equations. Therefore the advancement in systematics here is of particular usefulness.

The present paper is the extension and slightly different interpretation of my conceptions described in an issue of the journal *Prikladnaya Mekhanika i Tekhnicheskaya Fizika* devoted to the 80th anniversary of the first President of the Siberian Branch of the U.S.S.R. Academy of Sciences, M. A. Lavrentiev.

Intrinsic scales of uniform continua

Diffusion rates of mass, momentum and heat, and sound speed are the intrinsic scales determining the conditions of dynamic perturbation propagation in a given medium. The intensity of molecular diffusion is characterized by physical properties referred to as diffusivity, kinematic viscosity and thermal diffusivity of the same dimension, i.e.

$$\{D, \nu, a\} \left[\frac{M^2}{c}\right]. \tag{1}$$

From this it follows that if there exists a characteristic linear dimension δ, the scale of diffusion rate can be determined as

$$u_D = \frac{D}{\delta}. \tag{2}$$

In turn, provided that there exists some scale for the flow velocity U, one can construct a linear scale for the diffusional perturbation propagation

$$\delta_D = \frac{D}{u}. \tag{3}$$

This determines the Prandtl number interpretation

$$Pr = \frac{\nu}{a} \tag{4}$$

as a measure of the ratio of scales for the propagation of kinematic and thermal perturbations.

At the same time the above similarity number implies the specific physical property of the medium and divides all coolants into three classes as shown in Table 1.

In Table 1 δ and δ_T are the effective thicknesses of dynamic and thermal boundary layers, respectively.

In the uniform boundary layer theory the following intrinsic scales are known

dynamic velocity

$$\vartheta^* = \left(\frac{\tau}{\rho}\right)^{1/2} \tag{5}$$

displacement thickness

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_0 U}\right) dy \tag{6}$$

momentum thickness

$$\delta^{**} = \int_0^\infty \frac{\rho u}{\rho_0 U}\left(1 - \frac{\rho u}{\rho_0 U}\right) dy \tag{7}$$

energy thickness

$$\delta_e^{**} = \int_0^\infty \frac{\rho u}{\rho_0 U}\left(1 - \frac{T - T_0}{T_w - T_0}\right). \tag{8}$$

Scale of the wall viscosity region

$$\delta_\nu = \frac{\nu}{\vartheta_w^*} \tag{9}$$

and the time scale parameter of viscous relaxation processes

$$t^* = \frac{\nu}{\vartheta_w^{*2}}. \tag{10}$$

Similar scales can also be formed with the values δ^*, δ^{**} and δ_e^{**}.

From the parameters determining the viscous–gravitational interaction the linear scale

$$\delta_{\nu g} = \left(\frac{\nu^2}{g} \cdot \frac{\rho}{\Delta\rho}\right)^{1/3} \tag{11}$$

is formed.

Special linear field characteristics can be composed according to the relations of their derivatives, e.g. the mixing length scale by the Karman model

$$l \sim \frac{\partial^n u/\partial y^n}{\partial^{n+1} u/\partial y^{n+1}}. \tag{12}$$

The profound physical meaning of such scales has long been known in hydrodynamics as a universal law of velocity distribution in a turbulent boundary layer of incompressible liquid (Prandtl–Nikuradze)

$$\frac{u}{\vartheta_w^*} = f\left(\frac{\vartheta_w^* y}{\nu}\right) \tag{13}$$

Table 1

Medium	Metal liquids	Gases	Nonmetal liquids
Pr number	$\ll 1$	≈ 1	$\stackrel{\sim}{>} 1$
Ratio of boundary layer thicknesses	$\delta \ll \delta_h$	$\delta \approx \delta_h$	$\delta \stackrel{\sim}{>} \delta_h$
Heat transfer-friction relation	$St \nsim C_f$	$St \sim C_f$	$Pr \to \infty$ $St \sim C_f^{1/2}$

and a friction law in a laminar boundary layer on a plate

$$C_f\,Re^{**} = \text{constant.} \tag{14}$$

In equation (13) a natural velocity measure is its internal scale according to (5) and its distance from wall is measured by scale (9). Friction law constructed by the Reynolds number wherein the momentum thickness was introduced, is of the universal form adequate to the molecular friction mechanism. But if the Reynolds number is determined by the characteristic geometrical scale (for a circular tube it is radius R and for a plate it is length L), in the former case the friction law is

$$C_f\frac{UR}{\nu} = \text{constant} \tag{15}$$

and in the latter case

$$C_f\left(\frac{UL}{\nu}\right)^{1/2} = \text{constant} \tag{16}$$

i.e. in this system of similarity numbers they are not equivalent.

The intrinsic linear scale of the viscous–gravitational interaction is extremely useful in the thermodynamics of free-falling liquid films. But the heat transfer law under turbulent free thermogravitational convection in the unbounded liquid can also be interpreted as

$$\frac{\alpha}{\lambda}\delta_{vg} = \text{constant} \tag{17}$$

if in equation (11) instead of the value ν^2 the product νa for nonmetallic liquids and νa^2 for metallic ones are introduced. Here the self-similarity with respect to the external geometric size of the body and the decisive effect of the intrinsic linear scale of thermohydrodynamic perturbations are distinct.

For liquid films the reference hydrodynamic equation will be

$$C_f = f\left(\frac{\langle u'\rangle\langle\delta\rangle}{\nu'}\right) \tag{18}$$

which is slightly dependent on the wall geometric size.

The other important property of this flow is the existence of a wall layer in the laminar-wave regime which is quasi-self-similar relative to the flow rate, whose thickness is determined by the condition (Fig. 1)

$$\frac{\delta_1}{\delta_{vg}} \approx \text{constant.} \tag{19}$$

Acousto-dynamic interactions in uniform media is known to be characterized by the Mach number

$$M = \frac{U}{a_*} \tag{20}$$

where a_* is the adiabatic sound speed used here as the intrinsic scale of the threshold flow velocity.

In gases the similarity of fields of the physical properties which are essential for heat and momentum transfer, is sufficiently completely determined by the set of three 'temperature factors': the first temperature factor

$$\psi = \frac{T_w}{T_g} \tag{21}$$

the second temperature factor

$$\psi_s = \frac{C_s}{T_g} \tag{22}$$

where C_s is the Sutherland constant; and the kinetic temperature factor

$$\psi^* = 1 + r_*\frac{C_p - C_v}{2C_v}M^2 \tag{23}$$

where r_* is the recovery coefficient of temperature, C_p and C_v are the heat capacities at constant pressure and volume. As seen, here the constant C_s entering as the physical gas characteristics into the Sutherland formula for the temperature functions of transfer coef-

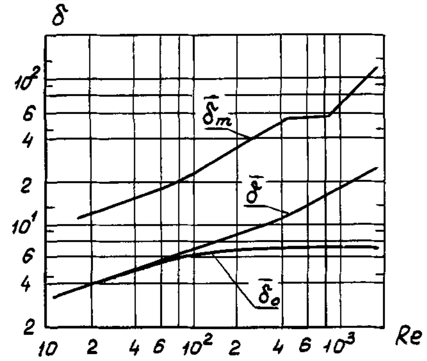

FIG. 1. Dependence of the dimensionless fluid wall layer thickness on the *Re* number. $\bar{\delta}_0$, thickness of the 'residual' wall layer; $\bar{\delta}$, mean thickness of the entire fluid layer; $\bar{\delta}_m$, maximum thickness of waves.

ficients, is some intrinsic scale of the dependence between the temperature field and the gas physical properties, and the Mach number characterizes the influence of the part of the temperature field which is determined by the medium motion.

INTRINSIC SCALES OF GAS–LIQUID SYSTEMS

In systems with interfaces there appear intrinsic scales associated with the interaction of external fields and surface effects. Due to the difference in phase densities, local effects of the Archimedes force and its interaction with the forces due to viscosity, surface tension, dynamic pressure, acoustic perturbations and so on take place.

Thus, for a vertical liquid film falling freely along a solid wall in a gas medium, linear scale (11) will be

$$\delta_{\nu g} = \left[\frac{\nu'^2}{g[1-(\rho''/\rho')]}\right]^{1/3}. \tag{24}$$

The linear scale of the capillary–gravitational interaction can be constructed as the Laplace number

$$\delta_{\sigma g} = \left[\frac{\sigma}{g(\rho'-\rho'')}\right]^{1/2}. \tag{25}$$

This value corresponds to the velocity scale of capillary–gravitational wave propagation

$$u_{\sigma g} = \left(\frac{g\sigma}{\rho'-\rho''}\right)^{1/4}. \tag{26}$$

When a new phase is generated on a surface (e.g. bubbling through a microporous surface; liquid boiling on the heater; vapour condensation on the cooling surface), the characteristic bubbling velocity can be derived from the flux density. Thus, the surface velocity if evaporation or condensation, is

$$U'' = \frac{q}{r\rho''} \tag{27}$$

and the momentum given to the liquid is

$$j''_w = \frac{q}{r}. \tag{28}$$

The linear scale characterizing a possibility for a new phase to generate is the minimum size of the nucleus at which the thermodynamic equilibrium is possible. Thus, for a vapour bubble nucleus

$$R_{\min} = \frac{2\sigma T''}{r\rho''\Delta T}. \tag{29}$$

Here T'' is the saturation temperature beneath the plane, ΔT is the initial superheating of liquid and r is the latent heat of evaporation.

INTRINSIC SIMILARITY NUMBERS OF GAS–LIQUID SYSTEMS

By introducing the intrinsic linear scales associated with the existence of interfaces, into the classical similarity numbers, it is possible to form dimensionless complexes free of any geometric flow characteristics. These similarity numbers are qualitatively new in characterizing the local interactions in the polyphase system and their influence on the integral parameters of the process.

For a gravitationally-viscous liquid film flow the Nusselt number can be written as

$$Nu^* = \frac{\alpha}{\lambda'}\left[\frac{\nu'^2}{g[1-(\rho''/\rho')]}\right]^{1/3} \tag{30}$$

and the flow Reynolds number as

$$Re_f = \int_0^{\delta}\frac{u'}{\nu'}\,\mathrm{d}y = \frac{G'}{\mu'}. \tag{31}$$

As is known from the coordinates, $\{Nu^*;\ Re_f\}$, experimental data on the heat transfer under film condensation of slowly moving pure nonmetallic vapours are practically unambiguously described. In this case the effect of the cooling surface geometry (vertical walls or horizontal tube bundles) is small, i.e. this dependence takes account of the main part of the law of film condensation of nonmetallic vapours.

The region of quasi-self-similar heat transfer, determined by the wall boundary layer in the laminar-wave regime of a falling liquid film, obeys the law

$$Nu^* \approx \text{constant} \tag{32}$$

which is identical to equation (17) for free turbulent thermogravitational convection.

After introducing a linear scale of the capillary–gravitational interaction into the Archimedes similarity number, one obtains the complex

$$Ar_* = \frac{\rho'^{1/2}\sigma^{3/2}}{\mu'^2[g[1-(\rho''/\rho')]]^{1/2}} \tag{33}$$

which characterizes the interaction of capillary, viscous and gravitational forces in the structure of gas–liquid mixtures. In particular, it influences the inception and development of gravitational–capillary waves in viscous liquids. Thus, according to Brauer's data, the inception of the laminar-wave flow of a vertical falling liquid film with the quasi-constant thickness of the residual wall boundary layer starts at

$$Re_f \approx 2.3\,Ar_*^{1/5} \tag{34}$$

and transforms into the fully developed turbulent flow at

$$Re_f \approx 35\,Ar_*^{1/5}. \tag{35}$$

Complex (33) can also be interpreted as some intrinsic Reynolds number characterizing the local capillary–gravitational viscous dynamic interactions and constructed by scales (25) and (26)

$$Ar_*^{1/2} = \frac{u_{\sigma g}\,\delta_{\sigma g}}{\nu'}. \tag{36}$$

For the bubbling and boiling heat transfer effective are the Nusselt and Peclet numbers constructed by linear scale (25) and momentum j''_w

$$Nu_* = \frac{\alpha}{\lambda'}\left[\frac{\sigma}{g(\rho' - \rho'')}\right]^{1/2} \tag{37}$$

$$Pe_* = \frac{C' j''_w}{\lambda'}\left[\frac{\sigma}{g(\rho' - \rho'')}\right]^{1/2}. \tag{38}$$

The criterion of the hydrodynamic gas–liquid structure stability is

$$k = \frac{U''_{cr}\,\rho''^{1/2}}{[\sigma g(\rho' - \rho'')]^{1/4}} \tag{39}$$

and it is the measure of the relation between dynamic pressure $\rho'' U''^2$ and the intrinsic scale of the Archimedes force $g(\rho' - \rho'')\delta_{\sigma g}$.

Experimental studies of the effect of blow-off and heat transfer under the developed nucleate bubbling through microporous surfaces distinctly indicate the non-uniformity of density variations of the gas bubbling liquid due to the pressure and molecular mass variations. This can be attributed to the capillary–gravitational acoustic interactions characterized by the original Mach number modification

$$M_* = \frac{u_{\sigma g}}{a_*}. \tag{40}$$

Here $a_* = (P/\rho'')^{1/2}$ is the isothermal sound speed in a gas phase.

It appears that the heating surface–liquid heat transfer under developed nucleate bubbling and boiling is determined by the complex

$$K_* = \frac{Pe_*}{M_*^2} \tag{41}$$

and approximately by

$$Nu_* \sim K_*^{2/3}. \tag{42}$$

Figure 2 illustrates the author's and Malenkov's experimental data on the developed nucleate bubbling heat transfer. The left-hand logarithmic line corresponds to the first (basic) region which is self-similar with respect to the liquid viscosity. The right-hand logarithmic line is the envelope of the subcritical (blow-off effect) region of the viscosity influence. These logarithmic lines embrace the region where the heat transfer is almost self-similar with respect to the dynamic similarity criterion (41).

As seen from Fig. 3, this heat transfer region under bubbling is, on the average, described by law (32). Hence there are several processes wherein visco-gravitational interaction leads to the rough self-similarity of the similarity criterion Nu^* based on the intrinsic visco-gravitational linear scale. In this case the values of the respective constants have the same order of magnitude:

(a) turbulent thermogravitational convection near a vertical plate $Nu^* \approx 0.13$;

(b) quasi-self-similar heat transfer region under laminary-wave regime of a vertical falling condensate film $Nu^* \approx 0.25$;

(c) heat transfer region which is quasi-self-similar with respect to the dynamics of bubbling through a microporous surface $Nu^* \approx 0.7$.

EFFECT OF FIRST-ORDER PHASE TRANSITION

In the case of phase transitions new phase generation can be set either by the specified heat flux density q, or by the specified difference between heating surface and saturation temperatures ΔT. In the former case the velocity of new phase generation is described by equation (27) and the dynamic similarity criteria are written in the canonical form or are the modification associated with the momentum introduction into equation (28). In the latter case the determining (independent) similarity criteria should contain the characteristic temperature difference, i.e. from the criteria involving the heat flux density one should separate the heat transfer coefficient α i.e. one or the other modification of the Nusselt number.

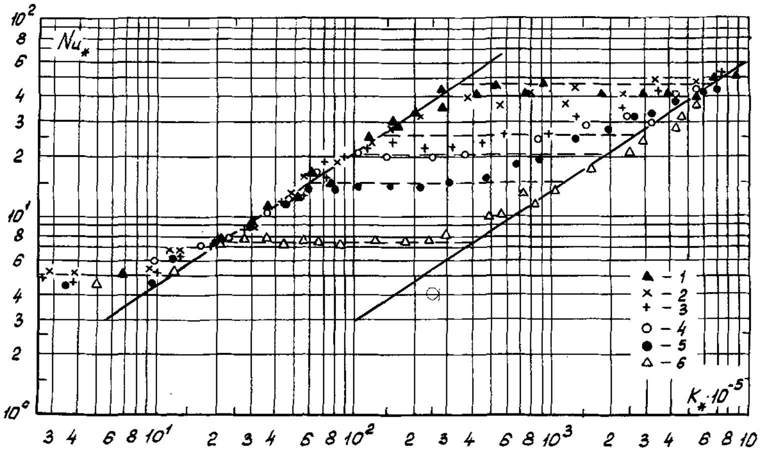

FIG. 2. Experimental data on heat transfer from a porous surface to water–glycerin solutions bubbled through by nitrogen and helium. Ar_*: 1, 1.26×10^3; 2, 1.21×10^3: 3, 0.865×10^3; 4, 0.0621×10^3; 5, 0.296×10^3; 6, 0.046×10^3.

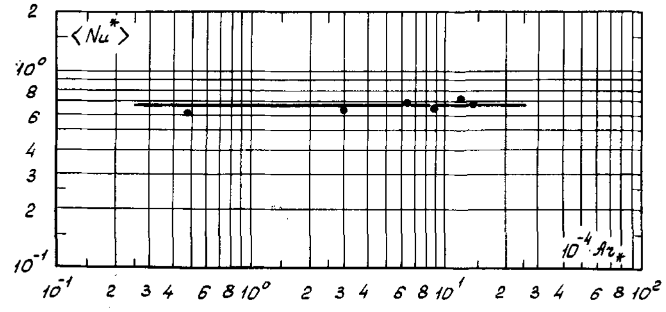

FIG. 3. Dependence of $\langle Nu^* \rangle$ on Ar_* in a self-similar heat-transfer region during bubbling of water and water–glycerin solutions.

Let two typical examples be given. In specifying q, for the film condensation of saturated vapour the determining similarity criterion is of the form

$$Re_f = \frac{qL}{r\mu'} \tag{43}$$

where L is the length of condensation path.

In specifying ΔT this criterion should be differentiated, e.g. as

$$Re_f = \frac{\alpha L}{\lambda'} \cdot \frac{a'}{\nu'} \cdot \frac{c'\Delta T}{r}. \tag{44}$$

Here the complex $(\alpha/\lambda')L$ is under determination (dimensionless function of the process), and the Prandtl number of the condensate and the thermal criterion of the phase transition

$$K = \frac{r}{c\Delta T} \tag{45}$$

are the complexes which are determining and dimensionless (the process arguments).

Under nucleate boiling, in the case of specifying q, the similarity criterion (41) is of the form

$$K_* = \frac{c'q p}{g\lambda' r} \tag{46}$$

and in the case of specifying ΔT, the complexes

$$\{Nu_*; K \cdot M_*^2\} \tag{47}$$

are separated. Phase transitions, in particular under evaporation and condensation, can change the regions of self-similarity, the nature of the effect of capillary–acoustic interactions, etc.

Figure 4 represents data on the developed nucleate boiling heat transfer as the dependence of the similarity criteria from (42) on the Archimedes number. The comparison with data on bubbling heat transfer (Fig. 2) indicates that the process of evaporation does not change the law (42) typical for the first self-similar region, but eliminates the viscosity effect in the studied range of Ar_*.

Figure 5 illustrates the dependence of the criterion

$$k = \frac{q_{cr,1}}{r\rho''^{1/2}[\sigma g(\rho' - \rho'')]^{1/4}} \tag{48}$$

on the criterion M_* for the first boiling crisis in bulk free-convection pool boiling of saturated liquid. It can

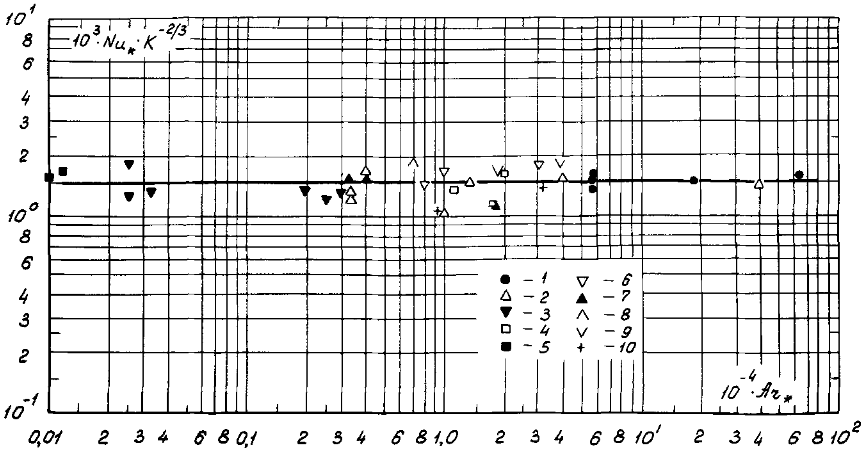

FIG. 4. Developed boiling heat transfer in the coordinates $\{Nu_* K^{-2/3}; A_*\}$ for different pressures: 1 water; 2 ethanol; 3 potassium; 4 sodium; 5 cesium; 6 cesium; 7 Freon-21; 8 Freon-12; 9 nitrogen; 10 helium; 11 benzol.

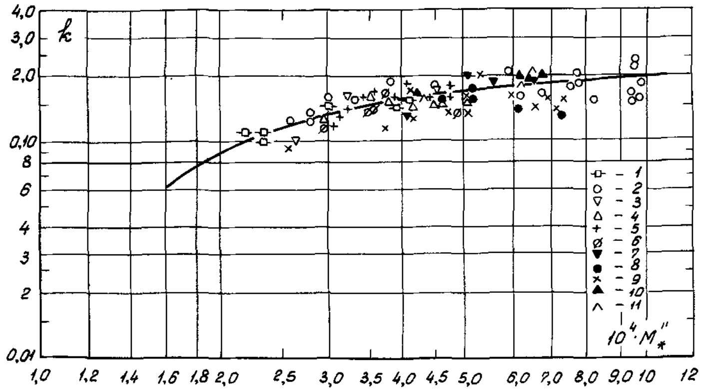

FIG. 5. The dependence $\{k; M_*\}$ according to the data for the first boiling crisis (transition from bubble to film boiling) during pool boiling of saturated liquid: 1 water (p = 1.0–190 × 10^5 N m^{-2}); 2 helium (0.06–2.07 × 10^5 N m^{-2}); 3 propane (21.5–34.6 × 10^5 N m^{-2}); 4 ethanol (0.06–42.2 × 10^5 N m^{-2}); 5 hydrogen (0.96–12.1 × 10^5 N m^{-2}); 6 methanol (1.0–63.5 × 10^5 N m^{-2}); 7 benzol (0.16–47.0 × 10^5 N m^{-2}); 8 oxygen (0.225–40.8 × 10^5 N m^{-2}); 9 nitrogen (0.413–29.8 × 10^5 N m^{-2}); 10 heptane (1.0–22.1 × 10^5 N m^{-2}); 11 pentane (2.0–31.5 × 10^5 N m^{-2}).

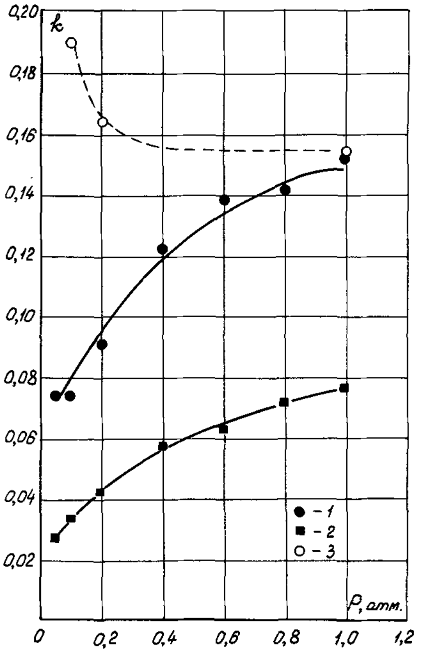

FIG. 6. Dependence of k on p for reduced (sub-atmospheric) pressures. Bubbling of nitrogen (1) and helium (2) through water; water boiling (3).

be noted that in a wide range of M_* the stability criterion of nucleate boiling remains practically constant within the spread of experimental points.

The phase transition is significantly changed by the effect of the conditions of gas and vapour bubbles generation on the stability criterion at low (as a rule, sub-atmospheric) pressures, as illustrated by the data in Fig. 6.

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ECHELLES INTRINSEQUES EN HYDROTHERMODYNAMIQUE DES GAZ

Résumé—On considère des échelles intrinsèques et des critères de similitude comme caractéristiques des interactions locales dans des écoulements de mélanges homogènes ou non. Dans l'analyse de similitude, une classe importante de valeurs est dégagée, dont des éléments ont été utilisés antérieurement comme une règle, à un niveau intuitif.

EIGENMASSSTÄBE DER THERMO-HYDRO-GASDYNAMIK

Zusammenfassung—Eigenmaßstäbe und Ähnlichkeitskriterien werden als charakteristisch für die lokalen Wechselwirkungen in Strömungen homogener und inhomogener Mischungen betrachtet. Dadurch wird in der Ähnlichkeitsanalyse eine wichtige Klasse von Werten getrennt formuliert, deren Elemente früher in der Regel intuitiv behandelt wurden.

СОБСТВЕННЫЕ МАСШТАБЫ В ТЕРМОГИДРОГАЗОДИНАМИКЕ

Аннотация — Рассматриваются собственные масштабы и числа подобия как характеристики локальных взаимодействий в потоках однородных и неоднородных сред. Тем самым в анализе подобия выделяется важный класс величин, элементы которого ранее использовались, как правило, на интуитивном уровне.